

STRATUS: AN INTERACTIVE STEADY STATE MIXED LAYER MODEL FOR PERSONAL COMPUTERS

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1. INTRODUCTION

We present here a steady-state, horizontally homogeneous, cloud-topped marine boundary layer model based primarily on the work of Lilly (1968) and Schubert et al. (1979). The conservative thermodynamic variables are equivalent potential temperature θ_e and total water mixing ratio $q + \ell$. Some of the differences between this and Lilly's (1968) model are: (1) radiation is allowed to penetrate into the boundary layer; (2) cloud top values of longwave radiation, equivalent potential temperature, and water vapor mixing ratio are linear functions of height derived from climatological data at California coastal stations; (3) the closure assumption assumes a weighted average of Lilly's (1968) maximum and minimum entrainment theories. This model has been programmed in FORTRAN and will run interactively on an IBM-compatible personal computer. The program allows the user to specify the geographical location, the wind speed, the sea-surface temperature, the large scale horizontal divergence, and the initial guess for cloud top height. Output includes the steady state values of cloud top and cloud base height, mixed layer equivalent potential temperature and total water mixing ratio, and the associated convective and radiative fluxes. The notation used throughout this abstract is that of Lilly with the exception that the subscript U replaces UH .

2. RADIATION PARAMETERIZATION

Since longwave cooling off cloud top is the important driving mechanism in the cloud-topped marine boundary layer, the radiation parameterization warrants a detailed description. We follow the philosophy that, consistent with the limitation of the vertical thermodynamic structure to two degrees of freedom, the vertical resolution of the radiative cooling should also be limited to two degrees of freedom. This means radiation can appear at most in the mixed layer thermal budget equation, and the cloud top jump condition. This is slightly more general than Lilly's restriction of the radiative cooling to the cloud top jump condition. Thus, we write the equations for the changes in radiative fluxes across cloud top and across the mixed layer as

$$F_U - F_H = (\rho c_p)^{-1} \left\{ (1 - \mu)(\sigma T_H^4 - \mathcal{L}_U^{\downarrow}) - (1 - \mu')S \right\}, \quad (1)$$

and

$$F_H - F_S = (\rho c_p)^{-1} \left\{ \mu(\sigma T_H^4 - \mathcal{L}_U^{\downarrow}) - \mu'S \right\}, \quad (2)$$

where ρ is the constant air density, σ is the Stefan-Boltzmann constant, T_H is the cloud top temperature in Kelvin, $\mathcal{L}_U^{\downarrow}$ is the downward longwave radiative flux attributed to that portion of the atmosphere which lies above the mixed layer, S is the absorbed broadband

shortwave radiative flux, and μ and μ' are the longwave and shortwave radiation partitions respectively. These partitions can take on values from zero to one. If both are set to zero, the radiation parameterization reduces to Lilly's case. Here and in the following sections the subscript U will refer to those properties of the upper air just above cloud top, the subscript H will refer to those properties just below cloud top, the subscript S will refer to those properties at the surface, and the subscript 0 will refer to those properties at some small height above the surface (typically taken to be 10 m). For this model, the downward longwave radiative flux term in (1) and (2) is expressed as a linear function of cloud top height. The function was derived based on the July average sounding data for the five years 1976–1980 as extracted from the *U.S. Dept. of Commerce Climatological Data, National Summary*. Two California locations were used, San Diego and Oakland. The sounding data combined with the midlatitude ozone profile as taken from the *U.S. Standard Atmosphere Table, 1976* and a uniform carbon dioxide profile of 0.501 g kg^{-1} was input into a broadband longwave radiation model written by Cox (1973). The model output was used to calculate the required linear relationships for the two locations. The resulting equations are given by

$$\mathcal{L}_U^\downarrow = \begin{cases} 314.0 + 0.03077H & \text{for Oakland,} \\ 333.1 + 0.32360H & \text{for San Diego,} \end{cases} \quad (3)$$

where the units for \mathcal{L}_U^\downarrow and H are Wm^{-2} and m respectively. The value of the absorbed solar radiation is that suggested by Lilly (1968), i.e. $S = 22.3 \text{ Wm}^{-2}$.

To calculate T_H , we start with the equation for potential temperature at cloud top (θ_H). The potential temperature at cloud top is equal to the potential temperature at cloud base (θ_h) plus the change that occurs when following a moist adiabat from cloud base to cloud top, i.e.

$$\theta_H = \theta_h + \frac{b}{a} \left(\frac{\alpha}{1 + \alpha} \right) (H - h). \quad (4)$$

In addition, θ_h can be calculated from our conservative thermodynamic variables by use of the definition of θ_e , i.e.

$$\theta_h = \theta_e - \frac{L}{c_p}(q + \ell). \quad (5)$$

To obtain the expression for the temperature at cloud top (T_H), we use both Poisson's equation and the integrated hydrostatic equation in potential temperature form. The resulting expression is

$$T_H = \left[\theta_h + \frac{b}{a} \left(\frac{\alpha}{1 + \alpha} \right) (H - h) \right] \left(\frac{p_H}{p_0} \right)^\kappa, \quad (6)$$

where p_H is determined hydrostatically from H .

3. THE COMBINED CONVECTIVE-RADIATIVE MODEL

With the discussion of the radiation parameterization completed, we can now write the combined convective-radiative model. The model equations, which are listed below, consist of two surface flux equations, the cloud base equation, the cloud top jump definitions, the cloud top temperature equation, the radiation equations, the consistency relation, the entrainment assumption, the mixed layer equivalent potential temperature and total water budget equations, and the cloud top jump condition on equivalent potential temperature.

These thirteen equations form a closed set in the thirteen unknowns H , h , θ_e , $(q + \ell)$, $\Delta\theta_e$, $\Delta(q + \ell)$, T_H , $(F_U - F_H)$, $(F_H - F_S)$, $(\overline{w'\theta'_e})_0$, $\overline{w'(q' + \ell')}_0$, $(\overline{w'\theta'_e})_H$, and $\overline{w'(q' + \ell')}_H$.

$$(\overline{w'\theta'_e})_0 = C_T V (\theta_{eS} - \theta_e) \quad (7)$$

$$\overline{w'(q' + \ell')}_0 = C_T V [q_S - (q + \ell)] \quad (8)$$

$$h = \frac{(1 + \alpha)(q_S - q_0) - a(\theta_{eS} - \theta_e)}{b} \quad (9)$$

$$\Delta\theta_e = \theta_{eU} - \theta_e \quad (10)$$

$$\Delta(q + \ell) = q_U - (q + \ell) \quad (11)$$

$$T_H = \left[\theta_h + \frac{b}{a} \left(\frac{\alpha}{1 + \alpha} \right) (H - h) \right] \left(\frac{p_H}{p_0} \right)^\kappa, \quad (12)$$

$$F_U - F_H = (\rho c_p)^{-1} \left\{ (1 - \mu)(\sigma T_H^4 - \mathcal{L}_U^\downarrow) - (1 - \mu')\mathcal{S} \right\}, \quad (13)$$

$$F_H - F_S = (\rho c_p)^{-1} \left\{ \mu(\sigma T_H^4 - \mathcal{L}_U^\downarrow) - \mu'\mathcal{S} \right\}, \quad (14)$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \overline{w'\theta'_e}_H \\ \overline{w'(q' + \ell')}_H \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad (15, 16)$$

$$\frac{\partial\theta_e}{\partial t} = \frac{(\overline{w'\theta'_e})_0 - (\overline{w'\theta'_e})_H - (F_H - F_S)}{H} \quad (17)$$

$$\frac{\partial(q + \ell)}{\partial t} = \frac{(\overline{w'q'})_0 - \overline{w'(q' + \ell')}_H}{H} \quad (18)$$

$$\frac{\partial H}{\partial t} = \frac{F_U - F_H - (\overline{w'\theta'_e})_H}{\Delta\theta'_e} - DH. \quad (19)$$

Equations (15) and (16) are simply a shorthand matrix notation for the consistency relation and the closure assumption. In the following section we discuss the steady-state solutions to the above model.

4. THE STEADY-STATE SOLUTIONS

In the steady-state case all derivatives with respect to time are set to zero, which results in a system of nonlinear algebraic equations. The method we have chosen to use in solving for the steady-state solutions is to reduce our system of equations to one equation in H . We can then use a simple secant method algorithm to iteratively find its zero. Before describing the steady-state model equations, it should be mentioned that the total water flux $\overline{w'(q' + \ell')}$ becomes constant with height. This can be seen directly from (18). For this reason, the subscripts on this variable are deleted.

The first step is to derive expressions for the surface fluxes of equivalent potential temperature and total water. The former is accomplished by eliminating the dependent variables θ_e and $(\overline{w'\theta'_e})_H$ between the steady-state forms of (7), (17) and (19). The resulting expression is

$$(\overline{w'\theta'_e})_0 = \frac{(F_U - F_H) + (F_H - F_S) + DH(\theta_{eS} - \theta_{eU})}{\left(1 + \frac{DH}{C_T V}\right)}. \quad (20)$$

The latter expression is derived similarly, resulting in

$$\overline{w'(q' + \ell')} = \frac{DH(q_S - q_U)}{\left(1 + \frac{DH}{C_{TV}}\right)}. \quad (21)$$

It should be noted that the cloud top values of equivalent potential temperature and water vapor mixing ratio (θ_{eU} and q_U respectively) are linear functions of height derived from the same climatological data as used in the radiation parameterization.

We can also eliminate θ_e from our cloud base equation. This is accomplished by substituting (7) and (8) into (9). The resulting expression is

$$h = \frac{(1 + \alpha)\overline{w'(q' + \ell')} - a(\overline{w'\theta'_e})_0}{bC_{TV}}. \quad (22)$$

To derive an expression for $(\overline{w'\theta'_e})_H$, we eliminate θ_e from (19) by using (7), which results in

$$(\overline{w'\theta'_e})_H = (F_U - F_H) - \frac{DH}{C_{TV}}(\overline{w'\theta'_e})_0 + DH(\theta_{eS} - \theta_{eU}). \quad (23)$$

Finally, we need to derive the steady-state closure equation. Before this can be done, however, it is necessary to provide expressions for the virtual potential temperature flux at the surface, just below cloud base, just above cloud base, and at cloud top, respectively. These expressions can be shown to take the form

$$(\overline{w'\theta'_v})_0 = (\overline{w'\theta'_e})_0 - \left(\frac{L}{c_p} - \bar{\theta}\delta\right)\overline{w'(q' + \ell')}, \quad (24)$$

$$(\overline{w'\theta'_v})_{h-} = (\overline{w'\theta'_e})_h - \left(\frac{L}{c_p} - \bar{\theta}\delta\right)\overline{w'(q' + \ell')}, \quad (25)$$

$$(\overline{w'\theta'_v})_{h+} = \left(\frac{1 + a\bar{\theta}(1 + \delta)}{1 + \alpha}\right)(\overline{w'\theta'_e})_h - \bar{\theta}\overline{w'(q' + \ell')}, \quad (26)$$

$$(\overline{w'\theta'_v})_H = \left(\frac{1 + a\bar{\theta}(1 + \delta)}{1 + \alpha}\right)(\overline{w'\theta'_e})_H - \bar{\theta}\overline{w'(q' + \ell')}, \quad (27)$$

where $(\overline{w'\theta'_e})_h$ is given by

$$(\overline{w'\theta'_e})_h = \left(1 - \frac{h}{H}\right)(\overline{w'\theta'_e})_0 + \frac{h}{H}(\overline{w'\theta'_e})_H. \quad (28)$$

With the above variables defined, we can now derive our closure equation. We use a weighted average of Lilly's (1968) maximum and minimum entrainment case to close our system equations. This closure equation takes the form

$$\frac{k}{H} \int_0^H \overline{w'\theta'_v} dz + (1 - k)(\overline{w'\theta'_v})_{\text{minimum}} = 0, \quad (29)$$

where k is a weighting parameter which can take on values in the range $0 \leq k \leq 1$. If we assume the daily averaged solar radiation is never strong enough to overcome the longwave

cooling and thus produce a net warming affect, the minimum virtual potential temperature flux must always be just below cloud base. This can be seen from (17). With this in mind, (29) can be integrated to give

$$\frac{2(1-k)}{k}(\overline{w'\theta'_v})_{h-} + (\overline{w'\theta'_v})_H + (\overline{w'\theta'_v})_{h+} + \frac{h}{H}[(\overline{w'\theta'_v})_S + (\overline{w'\theta'_v})_{h-} - (\overline{w'\theta'_v})_H - (\overline{w'\theta'_v})_{h+}] = 0. \quad (30)$$

We now have a closed set of equations (12)–(14), (20)–(28), and (30) in the unknowns H , h , T_H , $(F_U - F_H)$, $(F_H - F_S)$, $(\overline{w'\theta'_e})_H$, $(\overline{w'\theta'_e})_h$, $(\overline{w'\theta'_e})_0$, $w'(q' + \ell')$, $(\overline{w'\theta'_v})_H$, $(\overline{w'\theta'_v})_{h+}$, $(\overline{w'\theta'_v})_{h-}$, and $(\overline{w'\theta'_v})_0$. The method we have chosen to solve this system of equations can be explained in the following four step iteration sequence.

- (1) Make an initial guess of the cloud top height H and the cloud top jump in radiative flux $(F_U - F_H)$.
- (2) Using the current estimates of H and $(F_U - F_H)$, calculate in order $(\overline{w'\theta'_e})_0$, $w'(q' + \ell')$, h , $(\overline{w'\theta'_e})_H$ and the four virtual potential temperature fluxes using (20)–(27).
- (3) Again using the current estimate of H , calculate the new radiation variables T_H , $(F_U - F_H)$, and $(F_H - F_S)$. This newly calculated value of $(F_U - F_H)$ will be used in the next iteration if another iteration is required.
- (4) Using the above fluxes, check to see if (30) is satisfied to within some tolerable limit. If it is not, use the secant method to produce a new estimate of H and repeat steps (2)–(3) until (30) is satisfied.

5. RUNNING THE PROGRAM

The FORTRAN source code for the above model is stored on the attached floppy disk in the file STRATUS.FOR and the executable code in the file STRATUS.EXE. To run the program, simply enter STRATUS. You will be prompted to enter the sea-surface temperature, the large scale horizontal divergence, the wind velocity, and the initial guess for cloud top height. The output consists of cloud top and cloud base height, mixed layer equivalent potential temperature and total water mixing ratio, values of the convective and radiative fluxes and the cloud top temperature.

6. REFERENCES

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